

General Certificate of Education  
January 2005  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Further Pure 1**

**MFP1**

Tuesday 1 February 2005 Morning Session

**In addition to this paper you will require:**

- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables;
- an insert for use in Questions 7 and 8 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

**Information**

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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**1** The equation

$$x^2 - 5x - 2 = 0$$

has roots  $\alpha$  and  $\beta$ .

(a) Write down the values of  $\alpha + \beta$  and  $\alpha\beta$ . *(2 marks)*

(b) Find the value of  $\alpha^2\beta + \alpha\beta^2$ . *(2 marks)*

(c) Find a quadratic equation which has roots

$$\alpha^2\beta \quad \text{and} \quad \alpha\beta^2 \quad \text{span style="float: right;">*(3 marks)*$$

**2** A curve has equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

(a) Sketch the curve, showing the coordinates of the points of intersection with the coordinate axes. *(3 marks)*

(b) Calculate the  $y$ -coordinates of the points of intersection of the curve with the line  $x = 1$ . Give your answers in the form  $p\sqrt{2}$ , where  $p$  is a rational number. *(3 marks)*

(c) The curve is translated one unit in the positive  $x$  direction. Write down the equation of the curve after the translation. *(2 marks)*

**3** It is given that  $z = x + iy$ , where  $x$  and  $y$  are real numbers.

(a) Write down, in terms of  $x$  and  $y$ , an expression for  $z^*$ , the complex conjugate of  $z$ . *(1 mark)*

(b) Find, in terms of  $x$  and  $y$ , the real and imaginary parts of

$$2z - iz^* \quad \text{span style="float: right;">*(2 marks)*$$

(c) Find the complex number  $z$  such that

$$2z - iz^* = 3i \quad \text{span style="float: right;">*(3 marks)*$$

- 4 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a)  $\int_2^{\infty} 8x^{-3} dx$ ; (3 marks)

(b)  $\int_2^{\infty} (8x^{-3} + 1) dx$ ; (1 mark)

(c)  $\int_2^{\infty} 8x^{-3}(x + 1) dx$ . (3 marks)

- 5 (a) The transformation  $T_1$  is defined by the matrix

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Describe this transformation geometrically. (2 marks)

- (b) The transformation  $T_2$  is an anticlockwise rotation about the origin through an angle of  $60^\circ$ .

Find the matrix of the transformation  $T_2$ . Use surds in your answer where appropriate. (3 marks)

- (c) Find the matrix of the transformation obtained by carrying out  $T_1$  followed by  $T_2$ . (3 marks)

- 6 The angle  $x$  radians satisfies the equation

$$\cos\left(2x + \frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$$

- (a) Find the general solution of this equation, giving the roots as exact values in terms of  $\pi$ . (6 marks)

- (b) Find the **number** of roots of the equation which lie between 0 and  $2\pi$ . (2 marks)

7 [Figure 1, printed on the insert, is provided for use in this question.]

The variables  $x$  and  $y$  are known to be related by an equation of the form

$$y^3 = ax^2 + b$$

where  $a$  and  $b$  are constants.

Experimental evidence has provided the following approximate values:

$x$	1.5	4.0	5.0	6.5	8.0
$y$	5.0	6.3	7.0	8.0	9.0

(a) On **Figure 1**, draw a linear graph connecting the variables  $X$  and  $Y$ , where

$$X = x^2 \quad \text{and} \quad Y = y^3 \quad (5 \text{ marks})$$

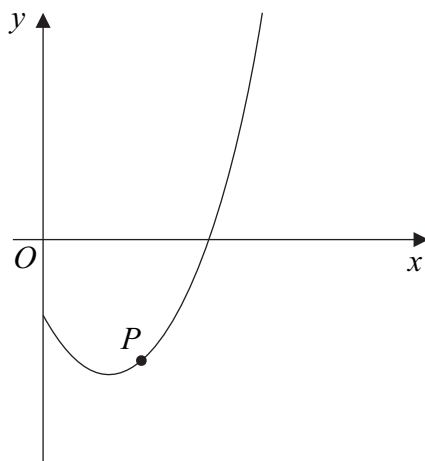
(b) From your graph, find approximate values for the constants  $a$  and  $b$ . (3 marks)

8 [Figure 2, printed on the insert, is provided for use in this question.]

The diagram shows a part of the graph of  $y = f(x)$ , where

$$f(x) = x^3 - 2x - 1$$

The point  $P$  has coordinates  $(1, -2)$ .



- (a) Taking  $x_1 = 1$  as a first approximation to a root of the equation  $f(x) = 0$ , use the Newton-Raphson method to find a second approximation,  $x_2$ , to the root. (3 marks)
- (b) On **Figure 2**, draw a straight line to illustrate the Newton-Raphson method as used in part (a).

Mark  $x_1$  and  $x_2$  on **Figure 2**. (2 marks)

- (c) By considering  $f(2)$ , show that the second approximation found in part (a) is not as good as the first approximation. (2 marks)
- (d) Taking  $x_1 = 1.6$  as a first approximation to the root, use the Newton-Raphson method to find a second approximation to the root. Give your answer to three decimal places. (2 marks)

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9 The function  $f$  is defined by

$$f(x) = \frac{x^2 + 2x + 2}{x^2}$$

(a) Write down the equations of the two asymptotes to the curve  $y = f(x)$ . *(2 marks)*

(b) By considering the expression  $x^2 + 2x + 2$ :

(i) show that the graph of  $y = f(x)$  does not intersect the  $x$ -axis; *(2 marks)*

(ii) find the non-real roots of the equation  $f(x) = 0$ . *(3 marks)*

(c) (i) Show that, if the equation  $f(x) = k$  has two equal roots, then

$$4 - 8(1 - k) = 0 \quad \text{span style="float: right;">*(3 marks)*$$

(ii) Deduce that the graph of  $y = f(x)$  has exactly one stationary point and find its coordinates. *(4 marks)*

**END OF QUESTIONS**

Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											

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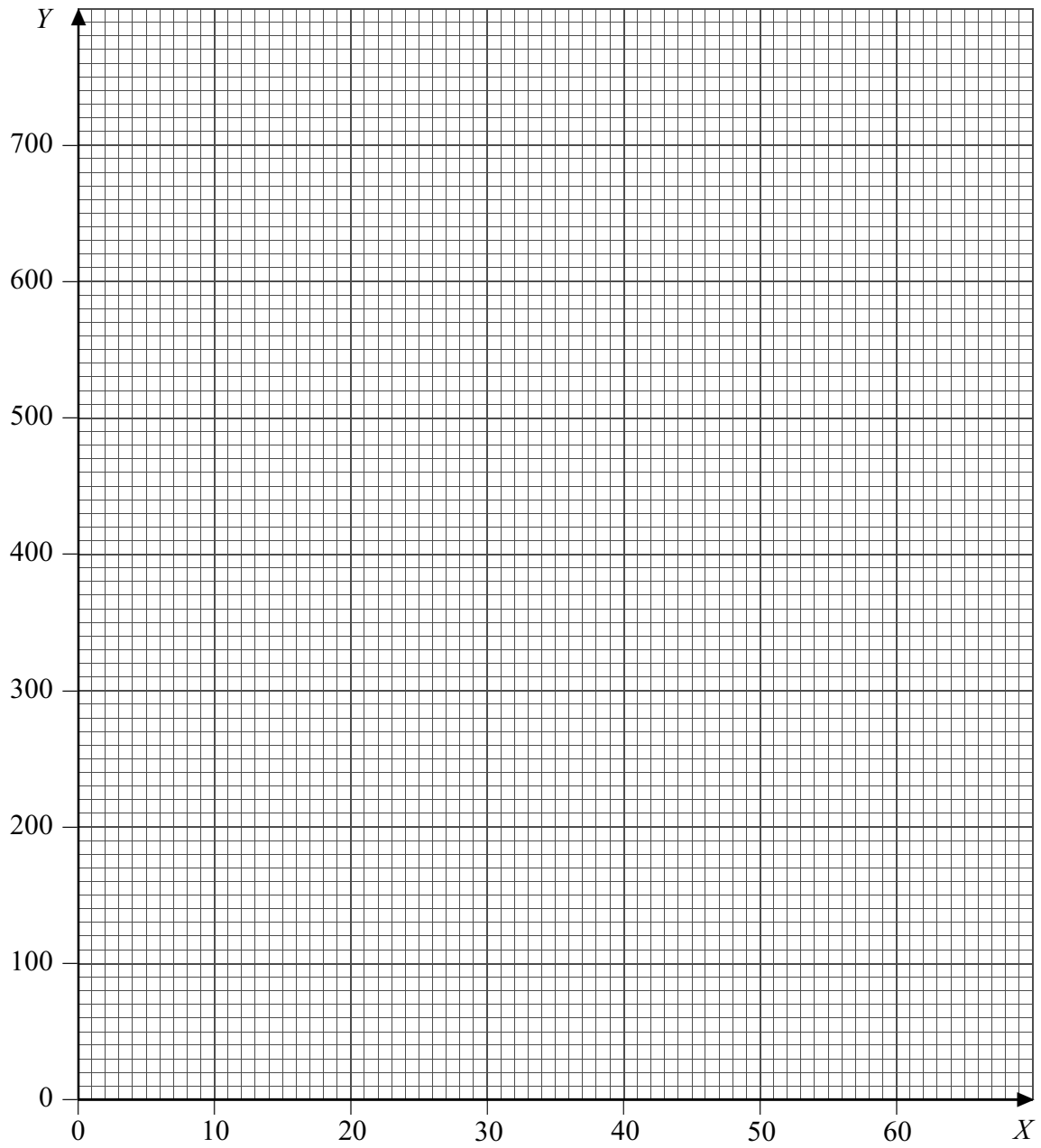
Insert for use in answering Questions 7 and 8.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

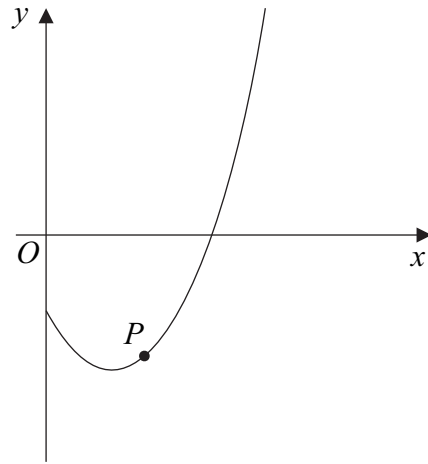
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**Figure 1** (for Question 7)





**Figure 2** (for Question 8)

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